USN | | | | | | | | | | | **14ELD11**

First Semester M.Tech. Degree Examination, June/July 2015

Advanced Mathematics

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

1 a. If $A = \begin{pmatrix} 5 & 9 \\ 1 & 7 \\ -3 & -5 \\ 1 & 5 \end{pmatrix}$. Find the QR factorization of A. **(08 Marks)**

b. Find the singular value decomposition of $A = \begin{pmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{pmatrix}$. **(12 Marks)**

2 a. Find the least-square solution of AX = B, given

$$A = \begin{pmatrix} 1 & 3 & 5 \\ 1 & 1 & 0 \\ 1 & 1 & 2 \\ 1 & 3 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 3 \\ 5 \\ 7 \\ -3 \end{pmatrix}$$

(08 Marks)

b. Obtain the pseudo-inverse of the matrix $A = \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$ using singular value decomposition. **(12 Marks)**

3 a. Prove that an extremal of the functional $I = \int_{x_1}^{x_2} \sqrt{y(1+(y')^2}\, dx$ is a parabola. **(08 Marks)**

b. Find the shortest distance between the parabola $y = x^2$ and the straight line $x - y = 5$. **(12 Marks)**

4 a. Find the extremal of the functional $I = \int_0^{\pi/4} [16y^2 - (y'')^2 + x^2]\, dx$ subject to the following end conditions: $y(0) = 0$, $y(\pi/4) = 1$, $y'(0) = 2$, $y'(\pi/4) = 0$. **(08 Marks)**

b. Prove that the sphere is the solid of revolution, which for a given surface area S, has maximum volume. **(12 Marks)**

5 a. An infinitely long string having one end at x = 0 is initially at rest on the x-axis. The end x = 0 undergoes a periodic transverse displacement described by $A_0 \sin \omega t$, $t > 0$. Using Laplace transforms, find the displacement of any point on the string at any time t. **(10 Marks)**

b. Using the Fourier cosine transform, find the temperature u(x, t) in a semi-infinite rod $0 \le x \le \infty$ determined by the partial differential equation $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$, $0 < x < \infty$, $t > 0$ subject to $u(x, 0) = 0$, $0 \le x \le \infty$, $u_x(0, t) = -u_0$ (a constant) when $x = 0$ and $t > 0$ u and $\frac{\partial u}{\partial x}$ both tend to zero as $x \to \infty$. **(10 Marks)**

6 a. State the properties of harmonic functions in bounded domains. **(06 Marks)**

b. Apply Fourier sine transform method to solve the following potential problem in the semi-infinite strip described by

$u_{xx} + u_{yy} = 0$, $0 < x < \infty$, $0 < y < a$

$u(x, 0) = f(x)$, $u(x, a) = 0$

$u(x, y) = 0$, $0 < y < a$, $0 < x < \infty$

$\frac{\partial u}{\partial x} \to 0$ as $x \to \infty$ **(14 Marks)**

7 a. Use Charne's penalty (Big-M) method to

Maximize $Z = 3x_1 + 2x_2$

Subject to the constraints $2x_1 + x_2 \le 2$

$3x_1 + 4x_2 \ge 12$

$x_1, x_2 \ge 0$ **(08 Marks)**

b. Apply dual simplex method to

Minimize $Z = 2x_1 + 2x_2 + 4x_3$

Subject to the constraints $2x_1 + 3x_2 + 5x_3 \ge 2$

$3x_1 + x_2 + 7x_3 \le 3$

$x_1 + 4x_2 + 6x_3 \le 5$

$x_1, x_2, x_3 \ge 0$ **(12 Marks)**

8 a. Use two-phase method to

Minimize $Z = \frac{15}{2}x_1 - 3x_2$

Subject to the constraints $3x_1 - x_2 - x_3 \ge 3$

$x_1 - x_2 + x_3 \ge 2$

$x_1, x_2, x_3 \ge 0$ **(12 Marks)**

b. Using Kuhn-Tucker conditions

Maximize $Z = 3x_1^2 + 14x_1x_2 - 8x_2^2$

$3x_1 + 6x_2 \le 72$, $x_1, x_2 \ge 0$ **(08 Marks)**

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